Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 6 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. (a) Use algebra to determine the values of x for which

$$\frac{5x}{x-2} \geqslant 12$$

(4)

(b) Hence, given that x is an integer, deduce the value of x.

(1)

(a) $$\frac{5x}{x-2} \geqslant 12$$

multiply by $(x-2)^2$ to ensure you're multiplying by a positive value, and hence the direction of the inequality stays the same

$$5x(x-2) \geqslant 12(x-2)^2 \quad (1)$$

$$5x^2 - 10x \geqslant 12x^2 - 48x + 48$$

$$7x^2 - 38x + 48 \leqslant 0 \quad (1)$$

$$(7x-24)(x-2) = 0$$

$\therefore$ critical values are $x = 2$, $x = \frac{24}{7}$ (1)

$x \neq 2$ ← $x-2$ cannot equal 0

$\therefore$ $2 < x \leqslant \frac{24}{7}$ (1)

(b) $x = 3$ ← 3 is the only integer in that range

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Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that the equation

$$3\cos x - 2\sin x = 1$$

can be written in the form

$$2t^2 + 2t - 1 = 0$$

(3)

(b) Hence solve, for $-180° < x < 180°$, the equation

$$3\cos x - 2\sin x = 1$$

giving your answers to one decimal place.

(4)

(a) $3\cos x - 2\sin x = 1$

Using derived t-formulae.

$\sin\theta = \frac{2t}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$3\left(\frac{1-t^2}{1+t^2}\right) - 2\left(\frac{2t}{1+t^2}\right) = 1$ (1)

$3(1-t^2) - 2(2t) = 1(1+t^2)$ (1)

$3 - 3t^2 - 4t = 1 + t^2$

$2t^2 + 2t - 1 = 0$ (1)

(b) $t = \frac{-2 \pm \sqrt{2^2 - 4\times 2\times -1}}{2\times 2}$ (1) ← using quadratic formula

$t = \frac{-1\pm\sqrt{3}}{2} \Rightarrow \frac{x}{2} = \tan^{-1}\left(\frac{-1\pm\sqrt{3}}{2}\right)$ (1)

$\frac{x}{2} = \tan^{-1}\left(\frac{-1+\sqrt{3}}{2}\right) \Rightarrow x = -107.6°$ (1)

$\frac{x}{2} = \tan^{-1}\left(\frac{-1-\sqrt{3}}{2}\right) \Rightarrow x = 40.2°$ (1)

Question 2 continued

(Total for Question 2 is 7 marks)

3. The rectangular hyperbola H has equation $xy = c^2$ where c is a positive constant.

The line l has equation $x - 2y = c$

The points P and Q are the points of intersection of H and l

(a) Determine, in terms of c, the coordinates of P and the coordinates of Q

(3)

The point R is the midpoint of PQ

(b) Show that, as c varies, the coordinates of R satisfy the equation

$$xy = -\frac{c^2}{a}$$

where a is a constant to be determined.

(2)

(a) $xy = c^2$ and $x - 2y = c \Rightarrow x = c + 2y$

$(c + 2y)y = c^2$ ① ← sub. second equation into the first

$2y^2 + yc - c^2 = 0$

$(2y - c)(y + c) = 0 \Rightarrow y = -c, \frac{c}{2}$ ①

When $y = -c$: $x = c + 2(-c) = -c$

When $y = \frac{c}{2}$: $x = c + 2\left(\frac{c}{2}\right) = 2c$

$\therefore$ points are $(-c, -c)$ and $\left(2c, \frac{c}{2}\right)$ ①

(b) Midpoint $= \left(\frac{-c + 2c}{2}, \frac{-c + \frac{c}{2}}{2}\right)$

$= \left(\frac{c}{2}, -\frac{c}{4}\right)$ ①

$xy = \frac{c}{2} \times -\frac{c}{4} = -\frac{c^2}{8}$ where $a = 8$ ①

Question 3 continued

(Total for Question 3 is 5 marks)

4. A teacher made a cup of coffee. The temperature θ°C of the coffee, t minutes after it was made, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} + 0.05(\theta - 20) = 0$$

Given that

- the initial temperature of the coffee was 95 °C
- the coffee can only be safely drunk when its temperature is below 70 °C
- the teacher made the cup of coffee at 1.15 pm
- the teacher needs to be able to start drinking the coffee by 1.20 pm

use two iterations of the approximation formula

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n \approx \frac{y_{n+1} - y_n}{h}$$

to estimate whether the teacher will be able to start drinking the coffee at 1.20 pm.

(6)

At $t = 0$, $\theta = 95 \Rightarrow t_0 = 0$ and $\theta_0 = 95$.

Temp. after 5 minutes, using 2 steps. $h = \frac{5}{2} = 2.5$ (1)

$\left(\frac{d\theta}{dt}\right)_0 = -0.05(95 - 20) = -3.75$ (1)

$\theta_1 \approx \theta_0 + h\left(\frac{d\theta}{dt}\right)_0$ ← rearrange the approximation formula for θ_2.

$\theta_1 \approx 95 + 2.5(-3.75) \approx 85.625$ (1)

$\left(\frac{d\theta}{dt}\right)_1 = -0.05(85.625 - 20) = -3.28125$ (1)

use value of θ_1

$\theta_2 \approx \theta_1 + h\left(\frac{d\theta}{dt}\right)_1$

$\theta_2 \approx 85.625 + 2.5(-3.28125) \approx 77.4$ (1)

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Question 4 continued

$\theta_2 = 77.4 > 70$ so the teacher will not be able to start drinking the coffee at 1:20 pm ①

(Total for Question 4 is 6 marks)

5. The points A, B and C are the vertices of a triangle.

Given that

- $\overrightarrow{AB} = \begin{pmatrix} p \\ 4 \\ 6 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} q \\ 4 \\ 5 \end{pmatrix}$ where p and q are constants

- $\overrightarrow{AB} \times \overrightarrow{AC}$ is parallel to $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

(a) determine the value of p and the value of q

(7)

(b) Hence, determine the exact area of triangle ABC

(2)

(a) $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ p & 4 & 6 \\ q & 4 & 5 \end{vmatrix}$ ← vector product formula ①

$= (4 \times 5 - 6 \times 4)i - (5p - 6q)j + (4p - 4q)k$

$= -4i + (6q - 5p)j + (4p - 4q)k$ ①

$\overrightarrow{AB} \times \overrightarrow{AC}$ is parallel to $r = 2i + 3j + 4k$, so $r \times (\overrightarrow{AB} \times \overrightarrow{AC}) = 0$

$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \times \begin{pmatrix} -4 \\ 6q - 5p \\ 4p - 4q \end{pmatrix} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -4 & 6q-5p & 4p-4q \end{vmatrix} = 0$ ①

$0 = [3(4p-4q) - 4(6q-5p)]i - [2(4p-4q) - 4(-4)]j$

$+ [2(6q-5p) - 3(-4)]k$

$0 = (32p - 36q)i - (8p - 8q + 16)j + (12q - 10p + 12)k$

↓ set each component equal to 0

(OR notice that $\overrightarrow{AB} \times \overrightarrow{AC} = -2(2i + 3j + 4k)$ and solve)

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Question 5 continued

$\therefore \quad 32p - 36q = 0$ ① for all 3 equations

$8p - 8q + 16 = 0 \Rightarrow p - q = -2$ ①

$12q - 10p + 12 = 0 \Rightarrow 6q - 5p = -6$ ②

solve simultaneously

① $\Rightarrow p = q - 2$

① into ② $\Rightarrow 6q - 5(q-2) = -6$ ①

$q + 10 = -6$

$q = -16$

$p = q - 2 = -16 - 2 = -18$

$\therefore p = -16, \; q = -18$ ① + ① for complete method

(b) $\overrightarrow{AB} \times \overrightarrow{AC} = -2(2i + 3j + 4k) = -4i - 6j - 8k$ ①

Area $= \dfrac{1}{2}\left|\begin{matrix}-4\\-6\\-8\end{matrix}\right| = \dfrac{1}{2}\sqrt{-4^2 + -6^2 + -8^2} = \sqrt{29}$ ①

formula for area of a vector triangle $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The parabola C has equation $y^2 = 4ax$ where a is a positive constant.

The point $P(at^2, 2at)$, $t \neq 0$, lies on C

The normal to C at P is parallel to the line with equation $y = 2x$

(a) For the point P, show that $t = -2$

(3)

The normal to C at P intersects C again when $x = 9$

(b) Determine the value of a, giving a reason for your answer.

(5)

(a) $y^2 = 4ax$ differentiate w.r.t. x

$2y\frac{dy}{dx} = 4a$ $\div 2y$

$\frac{dy}{dx} = \frac{2a}{y}$ 1

At P, $\frac{dy}{dx} = \frac{\cancel{2}a}{\cancel{2}at} = \frac{1}{t}$

Perpendicular gradient $m \times \frac{1}{t} = -1 \Rightarrow m = -t$ 1

Parallel to $y = 2x$ (gradient = 2) $\therefore -t = 2 \Rightarrow t = -2$ 1

(b) $P(at^2, 2at)$

$t = -2 \Rightarrow P((-2)^2a, 2(-2)a) = P(4a, -4a)$

Equation of normal: $y - y_1 = m(x - x_1)$

$y + 4a = 2(x - 4a)$ 1

$y = 2x - 12a$

$y^2 = 4ax \Rightarrow y = \sqrt{4ax}$ sub. into $y = 2x - 12a$

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Question 6 continued

$$\sqrt{4ax} = 2x - 12a$$

sub. in $x = 9$

$$4a(9) = (2(9) - 12a)^2$$

$$36a = 324 - 432a + 144a^2$$

$$144a^2 - 468a + 324 = 0 \quad ①$$

$\div 36$ (highest common factor)

$$4a^2 - 13a + 9 = 0$$

$$(4a-9)(a-1) = 0 \implies a = \frac{9}{4}, 1 \quad ②$$

Testing P with $a = \frac{9}{4}$.

$$P\left(\frac{9}{4}(-2)^2, 2\left(\frac{9}{4}\right)(-2)\right) = P(9;9) \neq P(4(1);-4(1))$$

Testing P with $a = 1$.

$$P\left(1(-2)^2, 2(1)(-2)\right) = P(4;4) = P(4(1);-4(1))$$

$\therefore a = 1 \quad ①$

Question 6 continued

(Total for Question 6 is 8 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS